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## *United States Expedition to Chili.*

(*Extract of a Letter from Lieut. G.M. Gilliss, U.S.N. to Admiral W.H. Smyth.*)

"Preparatory to the publication of our volume of *Differential Observations on Mars and Venus in Chili*, we are discussing our longitude, and need for this purpose all the corresponding moon-culminations in Europe.

"Congress acted very liberally in the order to publish our results, directing that the whole work should be well printed on good paper, of quarto size, and neatly bound. In all there will be five volumes. Vol. i., containing a full account of Chili (except its political history), and Lieut. MacRae's narrative of the magnetical expedition across the Andes and Pampas of Buenos Ayres, will fill more than 700 pages. It has 51 maps and plates of landscapes, natural history, &c. These are in the engraver's hands, and the volume will probably be ready in July.

"Vol. ii., the differential observations on *Mars* and *Venus* wherever made and furnished us. We shall probably fill 500 pages, and the volume will be put to press by May.

"Vols. iii. and iv., meridian observations and zones, each 500 to 600 pages. The instrumental and clock errors have been computed for a large portion of the time; but it is not probable the volumes will be printed in less than $2\frac{1}{2}$ years. The reduction of more than 20,000 *new* stars and their arrangement in the order of right ascension from the zones is a work of much time.

Vol. v., magnetical and meteorological observations, will fill also 500 or 600 pages. This will appear shortly after vol. ii.

"You will perceive that we three in Chili accomplished no small amount of labour; I hope the results may prove acceptable to men of science.

"*Washington, 8th Nov.*, 1854."

## *On the Resistance of the Ethereal Medium and the Attraction of the Small Planets.* By A. J. Angström, Esq., Junior Professor of Astronomy in the University of Uppsala.*

The planetary secular perturbations have more especially engaged the attention of astronomers, from the circumstance that the solution of questions, respecting the stability of our solar system,

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* This paper is a translation from the Swedish, by A. D. Wackerbarth, Esq. who also communicated it to the Society.

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finally depends on the amount of these variations, which, however inconsiderable they may appear, may, in the lapse of ages, accumulate to such a degree, as entirely to change the character of the elements.

Both theory and experience, however, agree that perturbations of such a nature cannot take place within our planetary system. The exception to this rule, which, in the beginning of the present century, was supposed to have been found in the moon's motion, whereof the mean motion had, from the earliest period to which astronomical observations extend, continually increased, disappeared when Laplace succeeded in explaining that phenomenon by a slow change in the eccentricity of the earth's orbit. Hence it appeared that this variation was, like its cause, periodical, and did not even affect directly the mean velocity, but what is called the epoch.

The case is, however, otherwise with comets. Here the largeness of the inclinations and eccentricities by no means excluded perturbations, even of a periodic nature, of such extent as entirely to change the elements of the orbit. A striking example of this is afforded by Lexell's comet, and, probably, also the two comets named after Brorsen and D'Arrest. And such must generally be the case with any comet that comes into the neighbourhood of *Jupiter.* But if the comets can be disturbed in their orbits by the planetary attractions, on the other hand, the masses of the comets are so trifling, that they cannot exercise any sensible influence on the motions of the planets.

Nevertheless, one of the comets, namely that of Encke, has given rise to a supposition which, should it be confirmed, must eventually make itself evident in the motions of the planets, and form a real exception to the received law of the unchangeableness of the mean distances, as subjected only to periodic variations.

It has, in fact, appeared that this comet — independently of the considerable changes which the period undergoes from the planetary perturbations—has, at each revolution, increased its mean velocity in its orbit, so as each time to arrive at perihelion $2\frac{1}{2}$ hours earlier than on the preceding occasion.

This circumstance is easiest explained in the following manner, according to the evidence produced by the learned calculator of that comet, Professor Encke:—there enters, or may be supposed to enter, into the expression for the mean longitude, a term varying as the square of the time; but then comes the difficulty of accounting for the presence of that term by the general law of gravitation. This has not hitherto been done; and Encke has accordingly had recourse to another means of explanation, namely, the resistance of an ethereal medium.

It is known that the effect which a resisting medium, of the same nature as the atmosphere, would have on a heavenly body's motion, would be to reduce the periodic time and eccentricity of the orbit, whereby there must arise an augmentation in the mean velocity, continually increasing with the time. This was just what was wanted; and, on the strength of this experience, Encke assumed, in speciality for his ether, that its density decreases as the square of the distance from the sun; and, furthermore, that the resistance is proportional to the density of the ether and the square of the comet's velocity.

This supposition is, however, subject to weighty objections. First and principally, if the density of the medium increases up to the surface of the sun, it must hence follow that the ether itself participates in the solar system's progressive motion, as well as in the sun's rotation; for we have no right to ascribe to the ether the properties of air in one instance, and deny the consequences thereof in another. The sun's rotatory motion is then immediately communicated to the circumjacent condensed ether, and is thence communicated from stratum to stratum, with either the same or a diminished angular velocity. If we assume that the angular velocity diminishes in the same ratio as that of the planets, it follows that the resistance to the planets and to the comets of direct motion is nearly evanescent, but is so much the more increased in the case of comets with retrograde motion. But that this is not the case is sufficiently evidenced by Halley's comet.

The apparent diminution in the volume of the nucleus of Halley's comet, when in the neighbourhood of the sun, has been taken as evidence of the ether's increasing density. But as ether possesses the property of passing freely through all bodies, an increase in its density would not compress, but, on the contrary, if it had any effect at all, would expand bodies. The Newtonian theory for the origin of comets' tails, namely, that they are turned from the sun for the same reason as vapours rise in our own atmosphere, Bessel* has shown to be insufficient to account for the various changes in their form.

Since, then, on the one hand, on the hypothesis of the ether's condensation, its resistance, in the case of Encke's comet, must vanish, and, on the other, that hypothesis is insufficient to explain the phenomena as desired, it would seem that it ought to be rejected.

On the density of ether within diaphanous bodies, opinions, as we know, differ; since some have considered that density as constant, while others again, like Fresnel, conceive it to be different in different media; but even if the second of these opinions be the more probable,† and thus the molecular forces have the power of altering the ether's density, yet assuredly no such power can be ascribed to gravity.

No traces of the resistance of ether are discoverable in the motions of the planets, and this has been considered as arising from their greater density and less volume; but this is subject to much

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* *Astron. Nach.* tome xiii. p. 185.

† See the Author's article on the "Plane of Polarisation." *Vetanskeps-Akademicus Förhandlingar*, 1853, No. 6.

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doubt. Fresnel's theory of aberration, as well as direct experiments by Fizeau,[*] seem to show that ether, to a certain degree, participates in a diaphanous body's motion, and that in proportion to the said body's refracting power. Thus, the ether's resistance takes place within the body itself, and must, therefore, be independent of the body's volume, but must increase in the ratio of its refracting power. But as this, on the other hand, is compensated by the greater mass which bodies of strong refracting power generally possess, it would seem that we are by no means at liberty to assume any great difference between planets and comets with respect to the ether's resistance.

Meanwhile our knowledge of the component parts of comets is extremely scanty. That they are not either solid or gaseous bodies, is evident from their property of not refracting light. They are, probably, of a powdery consistency. This we may conclude from their property of reflecting the sun-light, combined with the experience of certain observers,[†] that stars, seen through the nucleus itself, appear with even increased brightness. This last-mentioned phenomenon, may perhaps rest on an illusion, as others, for instance Bessel, have observed the contrary; but it may nevertheless be explained by the laws of the interference of light. For if a bright object be occulted by a body of small surface, that object may yet show itself with undiminished lustre behind the occulting body in the direction of the line of junction. Then if the occulting body be itself luminous, its light is added to that of the bright object, which is thus made to appear brighter.

As regards the assumption that the resistance varies as the square of the body's velocity, it is subject to the objection, that the notion of great or small can here only be applied to a velocity with reference to the medium in which the motion takes place; and thus that a velocity, which for air is considerable, cannot be so considered for ether. The speed with which these two separate media transmit undulations, is the best starting-point for a comparison. Thus it seems that the perihelion velocity of Encke's comet is barely one 5000th part of that of light, or about $\frac{1}{5}$ foot per second. This circumstance appears to me of especial weight, when one endeavours to reckon the resistance of ether, which again is still further modified by its probably arising within the body itself.

It would seem to follow, from the preceding considerations, that the density of the ether is throughout space the same, and that its resistance, if it really have any sensible influence, must vary as the first power of the velocity. Professor Hansen, who subjected the different hypotheses on the ether's resistance to a searching examination, has shown that the latter assumption makes but little difference in the numerical value of the resistance, but not so the

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[*] *Comptes Rendus*, xxxiii. p. 349.

[†] Piazzi, Struve, Reslhuber.

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former. For if we give the function (called by Encke U*) a value answering to the observed shortening of the periodic time, the perihelion distance decreases faster than it would if the ether's density increased in the neighbourhood of the sun, inasmuch as the eccentricity in that case diminishes more slowly. Encke has, indeed, with respect to the appearances of 1822 and 1832, when the comet was seen only after perihelion, uttered a suspicion, that the true law of the resistance is as yet undiscovered, and the deviations at the last appearance seem to confirm this: but whether the assumption of a constant density would represent the observations any better, cannot be decided beforehand. Still, to some extent, this would seem to be the case, since the observations of 1822 and 1832 tend to give the comet's orbit a greater eccentricity than what results when only observations before perihelion are used. They would, therefore, better harmonise with that hypothesis, as to the nature of ether, which makes the eccentricity decrease more slowly.

The objections to the hypothesis of the ether's resistance, which have been set forth in the preceding pages, have, however, awakened in my mind a lively wish to explain, by some other process, the acceleration which observations prove in the periodic time of Encke's comet; so much the more, as the celebrated proposer of that hypothesis himself remarks,—"dass das widerstehende Mittel nur als eine Form fur die nothwendig gewordene Correction anzusehen ist."†

The small planets, whose attractions have not hitherto been taken into account, have, in this respect, attracted my especial attention, and I here take the liberty of setting forth the results of the preliminary investigation that I have instituted on that subject. Even if it should appear that the observed inequality in the motion of Encke's comet cannot by this means be explained, yet every contribution to the knowledge of the mutual relations of these celestial bodies must possess at least some interest. Encke's comet will, assuredly, at some future time, throw light on the extent and masses of the small planets, and thus still further increase the number of truths wherewith it has already enriched astronomical investigations.

The number known of these bodies, from 4, which was the amount in the early part of the present century, has, within the last ten years, risen to 27. The rapidity with which these discoveries have followed one another, renders it probable that their real number is very great, although, on account of their smallness, the greater part of them are likely to remain unknown to us. Whether one adopt the Olbersian hypothesis, that all these asteroids are fragments of one large planet, or—be their origin what it may—that they form a sort of belt or ring

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* [U is used by Encke for the portion of the disturbing force that arises from the resisting medium.— A. D. W.]

† *Astr. Nachr.*, xii. p. 321.

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between *Mars* and *Jupiter*, it is evident that their secular perturbations, which are independent of their places in their orbits, must be the same, whether we consider them as all united in one, or disposed upon several points of a common orbit. The secular perturbations, then, of Encke's comet by the small planets, may give the approximate value of their aggregate mass.

To ascertain the mean position of the 27 asteroids' orbits, I have, from their elements, calculated the mean value of the following quantities :—

$$a\,e\cos.\pi, \qquad e\sin\pi, \qquad \sin\Omega\tan I, \qquad \cos\Omega\tan I,$$

and hence obtained,—

| For the Asteroids. | | For Encke's Comet. | |
|---|---|---|---|
| $\pi = 65^\circ\ 1'$ | $e = 0{\cdot}0555$ | $\pi_1 = 157^\circ\ 18'{\cdot}4$ | $e_1 = 0{\cdot}8478$ |
| $\Omega = 138\ 23$ | $a = 2{\cdot}567$ | $\Omega_1 = 334\ 29{\cdot}5$ | $a_1 = 2{\cdot}222$ |
| $I = 4\ 19$ | | $I_1 = 13\ 20{\cdot}7$ | |

where the elements of Encke's comet are, for comparison's sake, added. Moreover, if we call—

The inclination of the comet's orbit to that of the mean asteroid ......... $\iota$

The distance of the perihelion from the ascending node ..................... $\omega$

The same angle for the mean asteroid's orbit ................................ $\omega_1$

we have

$$\iota = 17^\circ\ 32' \qquad \omega = 186^\circ\ 47'{\cdot}5 \qquad \omega_1 = 94^\circ\ 21'$$

We see, then, that the two orbits form, as it were, two links in a common chain, and that their perihelia are nearly at right angles to one another. This, combined with the large value of $\iota$, shows that the two orbits cannot come into very close quarters with each other. On examining the asteroids' orbits separately, we find that, for some of them whose node-lines fall in the first, and perihelion in the second quadrant, an approach to the comet's orbit is possible. I have, for several of them, reckoned the angles $\omega$ and $\omega_1$, and hence the comet's and asteroids' radii-vectores in the line of nodes; but for only three has the difference been less than unity.

| | $\iota$ <br> ° ′ | $r$ | $r'$ | Diff. |
|---|---|---|---|---|
| Irene ......... | 18 48 | 2·361 | 2·811 | 0·450 |
| Proserpine ... | 15 52 | 3·192 | 2·662 | 0·527 |
| Ceres............ | 18 48 | 2·023 | 2·959 | 0·936 |

where $r$ is the comet's, and $r'$ the asteroid's radius-vector, in the line of the comet's ascending node. Hence we see that among the hitherto discovered asteroids, there is none that approaches sufficiently near to the comet's orbit to exercise any sensible influence on the comet's motion. It does not, however, hence

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follow that there may not be such among the yet undiscovered. Were there one with a mean distance of nearly 2·222, it would produce a periodic inequality of a secular nature, which, in spite of the asteroids' smallness, might be sensible on the integration of the differential formulas.

To return to the secular perturbations which the asteroids cause to Encke's comet. This belongs to the more difficult class of astronomical problems, inasmuch as the projections of the two orbits on the ecliptic intersect each other. Bessel* has indeed shown how to find the secular perturbations which a planet causes to a comet, or planet with a comet-like orbit, when the disturbing force can be developed in a converging series in powers of $\frac{r'}{r}$, if $r$ and $r'$ signify the radii-vectores of the comet and planet respectively; and Hansen† has discussed the opposite case in which the comet's radius-vector is always less than the planet's; but these methods both fail when the orbits cross one another.

The following method will not, then, be devoid of interest, although it must be owned that, when the mean distances of the two bodies are nearly equal, it does not lead to very rapidly converging series.

If we designate by $m$ and $m'$ the comet's and disturbing planet's respective masses, and their co-ordinates by

$$x, y, z, r, x', y', z', r',$$

and their distance from each other by $\varrho$, we obtain, as is well known, the components of the disturbing forces by differentiating the Lagrangian function,

$$\mathrm{R} = -m' . \frac{(x . x' + y . y' + z . z')}{r'^3} + \frac{m'}{\varrho};$$

but as the secular perturbations are independent of the planet's co-ordinates, we can reject the first term of this expression, whence we have

$$\mathrm{R} = \frac{m'}{\varrho}.$$

Since, moreover, in this expression those quantities only are to be retained which are independent of the planet's mean anomaly $n' t$, we may put

$$\mathrm{R}_1 = \frac{m'}{2\pi} \int_0^{2\pi} \frac{dn' t}{\varrho} \ddagger$$

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* *Astron. Nachr.*, xiv., p. 1.

† *Ermittelung der absoluten Störungen in Ellipsen*, 1843.

‡ [If we develope $\frac{m'}{\varrho}$ in a series of multiples of the cosines of the mean heliocentric elongation, and then cast away all the terms involving $n' t$, there will evidently remain only the series of constants expressing this definite integral.—A. D. W.]

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For the finding of this integral it is necessary to expand $\frac{1}{\varrho}$ in a converging series. For this purpose, if we call the angle between the radii-vectores U, we have

$$\varrho = \sqrt{(r^2 + r'^2 - 2 . r . r' . \cos U)},$$

and can put

$$\frac{1}{\varrho} = \frac{1}{(r^2 + r'^2)^{\frac{1}{2}}} + \frac{1}{2} . \frac{(2 . r . r' . \cos U)}{(r'^2 + r^2)^{\frac{3}{2}}} + \frac{1 . 3}{2 . 4} . \frac{(2 . r . r' . \cos U)^2}{(r'^2 + r^2)^{\frac{5}{2}}} + \ldots \quad (a)$$

or generally,

$$\frac{1}{\varrho} = \frac{1}{(r'^2 + r^2)^{\frac{1}{2}}} + \Sigma \frac{1 . 3 . 5 \ldots (2 . n - 1)}{2 . 4 . 6 \ldots (2 . n)} . \frac{(2 . r . r' . \cos U)^n}{(r'^2 + r^2)^{n + \frac{1}{2}}}$$

where $n$ takes all positive values from 1 to ∞.

The convergency of this series is determined by the expression $\frac{2 . n - 1}{2 . n} . \frac{2 . r . r' . \cos U}{r'^2 + r^2}$, which must always be less than unity, if $r$ is either greater or less than $r'$, and also even if $r = r'$, unless at the same time $\cos U = 1$. The series $(a)$ may, therefore, be considered as always convergent, except in the case of a collision between the two celestial bodies, in which case, however, the differential equations also would cease to hold.

Since, for perturbations of the first order, if $u'$ be the excentric anomaly

$$d . n' t = \frac{r'}{a'} . d u'$$

we have $$\frac{1}{2 . \pi} . \int_0^{2\pi} \frac{d . n' t}{\varrho} = \frac{1}{2 . \pi} . \int_0^{2\pi} \frac{r'}{a' . \varrho} . d u' = \frac{R_1}{m'} .$$

It is not necessary for the determination of $R_1$ to avoid developments proceeding according to the ascending powers of the planet's excentricity $e'$. If, then, we put

$$\frac{1}{m'}, R_1 = \frac{V^{\circ}}{(a'^2 + r^2)^{\frac{1}{2}}} + \frac{V^{i}}{(a'^2 + r^2)^{\frac{3}{2}}} + \frac{V^{ii}}{(a'^2 + r^2)^{\frac{5}{2}}} + \frac{V^{iii}}{(a'^2 + r^2)^{\frac{7}{2}}} + \&c. \ldots (b)$$

and reject terms multiplied by $e'^3$ and the higher powers, we have

$$V^{\circ} = 1$$

$$V^{i} = -\frac{2}{3} . a' . e' . p - \frac{3}{4} . a'^2 . e'^2$$

$$V^{ii} = \frac{3}{4} . a'^2 . r^2 . \{(p^2 + q^2) + e'^2 . (4 . p^2 - q^2)\} + \frac{3}{2} . a'^3 . r . e' . p + \frac{3}{4} . a'^4 . e'^2$$

$$V^{iii} = \frac{-75}{16} . a'^3 . r^2 . e' . p . (p^2 + q^2) - \frac{15}{32} . a'^4 . r^2 . e'^2 . (25 . p^2 + 2 . q^2)$$

$$V^{iiii} = \frac{105}{64} . a'^4 . r^4 . (p^2 + q^2)^2 + \frac{1}{4} . a'^4 . r^4 . e'^2 . (18 . p^4 + 15 . p^2 . q^2 - 3 . q^4)$$
$$+ \frac{105}{64} . a'^5 . r' . e' . p^3 . (p^2 + q^2)$$

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and so on, or if we preserve only the first power of $e'$,

$$\left.\begin{aligned}\frac{1}{m'},\ R_1 = {} & \frac{1}{(a'^2+r^2)^{\frac{1}{2}}} + \frac{1.3}{2.4}.\frac{1}{2}.\frac{a'^2.(p^2+q^2).2^2.r^2}{(a'^2+r^2)^{\frac{5}{2}}} + \frac{1.3.5.7}{2.4.6.8}.\frac{1.3}{2.4}.\frac{a'^4.(p^2+q^2)^2.2^4.r^4}{(a'^2+r^2)^{\frac{9}{2}}} \\ & + \frac{1.3.5.7.9.11}{2.4.6.8.10.12}.\frac{1.3.5}{2.4.6}.\frac{a'^6.(p^2+q^2)^3.2^6.r^6}{(a'^2+r^2)^{\frac{13}{2}}} + \&c. \quad . \quad . \quad . \\ & - \frac{1}{2}\ \frac{3}{2}.\frac{a'.e'.p.2.r}{(a'^2+r^2)^{\frac{3}{2}}} - \frac{1.3.5}{2.4.6}.\frac{3.5}{2.4}.\frac{a'^3.e'.p.(p^2+q^2).2^3.r^3}{(a'^2+r^2)^{\frac{7}{2}}} \\ & - \frac{1.3.5.7.9}{2.4.6.8.10}.\frac{3.5.7}{2.4.6}.\frac{a'^5.e'.p.(p^2+q^2)^2.2^5.r^5}{(a'^2+r^2)^{\frac{11}{2}}} - \&c. \quad . \quad . \quad . \\ & + \frac{3}{2}.\frac{1}{2}.\frac{a'^3.e'.p.2.r}{(a'^2+r^2)^{\frac{5}{2}}} + \frac{3.5.7}{2.4.6}.\frac{1.3}{2.4}.\frac{a'^5.e'.p.(p^2+q^2).2^3.r^3}{(a'^2+r^2)^{\frac{9}{2}}} \\ & + \frac{3.5.7.9.11}{2.4.6.8.10}.\frac{1.3.5}{2.4.6}.\frac{a'^5.e'.p.(p^2+q^2)^2.2^5.r^5}{(a'^2+r^2)^{\frac{13}{2}}} + \&c \quad . \quad . \quad . \end{aligned}\right\} (c)$$

where the law of the development is evident.

In the foregoing expression $p$ and $q$ have the following meaning:—

$$rp = \alpha x - \beta y \qquad x = a.(\cos u - e)$$

$$rq = -\gamma x + \delta y \qquad y = a.\sqrt{1-e^2}.\sin u.$$

$$\alpha = \cos\omega'.\cos\omega + \sin\omega.\sin\omega'.\cos\iota,$$

$$\beta = \cos\omega.\cos\omega' - \cos\omega.\cos\omega'.\cos\iota,$$

$$\gamma = \sin\omega'.\cos\omega - \cos\omega'.\sin\omega.\cos\iota,$$

$$\delta = \sin\omega.\sin\omega' + \cos\omega.\cos\omega'.\cos\iota,$$

where, as before, $\iota$ denotes the inclination of the comet's to the planet's orbit, and $\omega$ and $\omega'$ the distance of the respective orbits' the perihelia from comet's ascending node in the planet's orbit.

In order to obtain the secular variations of the elements we must differentiate $R_1$ with respect to $\omega$, $\omega'$, $e$, and $\iota$, and then integrate the result so obtained with respect to the time $t$. Thus, for example,

$$\delta e = -\frac{a.n}{e}.\sqrt{1-e^2}.\int(A_0 + A_1.\cos u + A_2.\cos 2u + \&c. ...).dt$$

where

$$A_0 = \frac{1}{2.\pi}.\int_0^{2\pi}\frac{dR_1}{d\omega}.du, \qquad A_2 = \frac{1}{2.\pi}.\int_0^{2\pi}\frac{dR_1}{d\omega}.\cos u.du, \ \&c.$$

These last integrals are obtained by the application of elliptic functions. For putting

$$r = a.(1-e.\cos u), \text{ and } (\tan\tfrac{1}{2}.u)^2 = \frac{\alpha}{\beta}.(\tan\tfrac{1}{2}.\psi)^2,$$

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where $\alpha^2 = \frac{a'^2 + a^2.(1-e)^2}{a'^2+a^2}$, and $\beta^2 = \frac{a'^2 + a^2.(1+e)^2}{a'^2+a^2}$,

we get

$$\int_0^{2\pi *} \frac{du}{(a'^2+r^2)^{\frac{2n+1}{2}}} = \frac{k}{\sqrt{\alpha}\,.\,\beta\,.\,(a'^2+a^2)^{\frac{2n+1}{2}}}\,.\int_0^{2\pi} \frac{(1+\mu\,.\cos\psi)}{\{1-c^2.(\sin\psi)^2\}^{\frac{2n+1}{2}}}.d\psi..(d)$$

where $k = \frac{\alpha+\beta}{2\alpha\beta}$, $\mu = \frac{\beta-\alpha}{\beta+\alpha}$, $c^2 = \frac{(\alpha+\beta)^2-4}{4\alpha\beta}$.

We have, moreover,—

$$\cos u = \frac{\mu + \cos\psi}{1+\mu\cos\psi} \quad\text{and}\quad \sin u = \frac{\sqrt{1-\mu^2}\,.\sin\psi}{1+\mu\cos\psi}$$

$$\text{and if}\quad 1+e\,.\mu = f \quad\text{and}\quad e-\mu = g,$$

$$1 - e\cos u = \frac{f+g\,.\cos\psi}{1+\mu\,.\cos\psi}; \qquad \cos u - e = \frac{f.\cos\psi - g}{1+\mu\,.\cos\psi}$$

The integral, in the right-hand member of equation (*d*), may, as is known, be expressed by elliptic functions. Furthermore, since the value of $\frac{d\,R_1}{d\,\omega}$, and the other differentials of $R_1$, are composed of terms, all having the form,—

$$\frac{B}{(a'^2+r^2)^{\frac{2n+1}{2}}},$$

in which

$$B = f(\cos u, \sin u, a, e)$$

it is easily seen that they may all be expressed by elliptic functions.

To the numerical application of the foregoing, I shall have the honour of returning on a future occasion; and for the present, therefore, confine myself to the following remarks.

In the value of the mean longitude, a term, varying as the square of the time, can arise only from the perturbations of the second order, which belong to the epoch. For if in the perturbations of the epoch of the first order, which vary proportionally to the time, we insert the variations of the elements, which are also proportional to the time, there must necessarily arise a term, varying as the square of the time.

I find, by an approximate calculation, that the variation of the eccentricity of Encke's comet, as far as it is caused by the influence of the asteroids, must, for the present, be negative; so that

$$\delta e = -t \times \text{constant.}$$

There arises, mainly through the action of *Jupiter* and *Saturn*, in the value of the epoch, a term proportional to the time, to be

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* Mr. Wackerbarth, in a note which we regret we have not room to insert here, has given the analysis of the process by which this transformation is effected.—Editor.

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added to the comet's mean anomaly $n\,t$. If in this the value of $\delta\,e$ be inserted, there must result a term diminishing with the square of the time. Whether, however, that term be of sufficient amount to account for the observed circumstances in the comet's period, is another question, which mainly depends upon the value we assign to the asteroids' masses and the eccentricity of the assumed common orbit. Against this, it may be urged, that observation does not indicate any very considerable change in the eccentricity of the comet's orbit; and, moreover, that the value of the asteroids' masses cannot unconditionally be assumed greater than what answers to their influence on the nearest planets.* But, on the other hand, it is not necessary to ascribe to that cause alone the whole of the observed diminution of the period, since that can partially be explained by the perturbations of the first order, which the asteroids cause to the period or epoch ; and besides, the elements of the orbit themselves must be somewhat altered if the ether's resistance be ignored. The necessity of a correction for the period of Encke's comet may, indeed, be considered as fully established by observation, but not so the form of that correction. For as far as that proof rests upon the apparitions of 1805, 1795, and 1786, the observations are better satisfied by assuming a periodic correction, embracing about seventy-two revolutions of the comet ; though, it must be owned, that this does not in any way facilitate the explanation of the phenomenon.

In the foregoing investigation, I have assumed that the resisting medium is the same ether, whose existence is presupposed as necessary for explaining the phenomena of light and heat : one might also suppose that the resisting medium is the last stratum of the sun's atmosphere, which, by its reflexion, probably causes the so-called "corona" in a total eclipse. It happens, however, in this case, that that atmosphere must participate in the sun's rotation, and thus its resistance must be insensible.

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* Since the above was written, the Author has had the opportunity of examining, in the *Comptes Rendus*, Nov. 28, 1853, an interesting article, by M. Le Verrier, on the masses of the small planets. Le Verrier considers that, with regard to the relative positions of the common asteroidal orbit, and *Mars'* orbit, the second term of the series, expressing the motion of *Mars'* perihelion, rises to $\frac{1}{4}$ of the first term of the same series. If, then, under this supposition, we take the aggregate mass of all the small planets as equal to the earth's mass, they must, according to M. Le Verrier, cause an inequality in the heliocentric longitude of *Mars'* perihelion, amounting to 11′ in a century. Now, as an inequality of one fourth of that amount could not assuredly have escaped the notice of observers, M. Le Verrier concludes that the sum of the masses of the small planets cannot surpass $\frac{1}{4}$ of the earth's mass. Even if with time this fraction should undergo some considerable modification, still a possibility is thus afforded for determining the limit of the masses of the small planets, which cannot but be of the highest utility in the question of the influence of these masses on Encke's comet.

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The following communication relative to the elements of Comet II. 1854, has been received from Mr. Powell. It will be seen that it includes the application of the corrections for aberration and parallax, which were not taken into account in his previous investigations.

## *Correction of the Elements of Comet II.* 1854.

By E. B. Powell, Esq.

| Greenwich M.T. | Long. Observed, Corr. for Aberr. and Parallax. | Corr. Applied. | Lat. Observed, Corr. for Aberr. and Parallax. | Corr. Applied. |
|---|---|---|---|---|
| April | | | (South) | |
| | ° ′ ″ | | ° ′ ″ | |
| 8·06067 | 49 51 43 | +47 | 5 22 57 | +33 |
| 13·087693 | 59 0 44 | +41 | 12 33 45 | +26 |
| 27·08739 | 75 31 8 | +31 | 24 29 4 | +16 |

The sun's real Long. was used, instead of that in the *Nautical Almanac.*

"The aberration was obtained from the diurnal motion and distance of the comet by means of a table in Vince's Astronomy; and the parallax was found roughly by taking the height and longitude of the nonagesimal from a large celestial globe, and then applying calculation.

"On pushing further the approximations, I found

$D_{,}$ the perihelion distance = ·2773579

$E_{,}$ the time of perihelion passage = March 24·010625

which afforded the errors of anomaly,

$u-\nu = +\ 1''$, from 8th to 13th, in an angle of 10° 41′

$u'-\nu' = -16''$, from 8th to 27th, in an angle of 27 11

"If D be diminished by ·00003, the errors become

$$u-\nu = +6'' \quad \text{and} \quad u'-\nu' = -6''$$

and therefore less on the whole; but I have no doubt of the observation of the 27th being more inaccurate than those of the 8th and 13th; and, consequently, I am inclined to prefer the value of D first mentioned.

"The resulting elements are

D = ·2773579

E = March 24th, ·010625, or 24th, $0^h\ 15^m{\cdot}3$

Inclination of Orbit = 82° 26′ 43″

Long. ☊ = 315 30 11

Long. ϖ on ecliptic = 167 56 19

Motion retrograde.

"Working out the comet's place on the 18th,

The error of long. = about 10″

The error of lat. = about 4